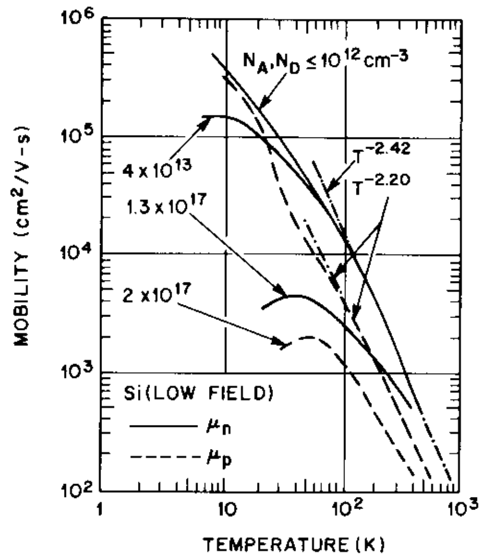

Fig. 19 Mobility of electrons and holes in Si as a function of temperature. (After Jacoboni et al., Ref. 37.)

For nondegenerate semiconductors, that is, where n is much smaller than N_C, Eq. 43a reduces to

$$D_n = \left(\frac{kT}{q}\right)\mu_n \tag{44a}$$

and similarly

$$D_p = \left(\frac{kT}{q}\right)\mu_p. \tag{44b}$$

Equations 44a and 44b are known as the Einstein relationship. At 300 K $kT/q = 0.0259$ V, and values of D are readily obtainable from the mobility results shown in Fig. 18. The mobilities discussed above are the conductivity mobilities, which have been shown to be equal to the drift mobilities.[27] They are, however, different from the Hall mobilities considered in the next section.

1.5.2 Resistivity and Hall Effect

The resistivity ρ is defined as the proportionality constant between the electric field and the current density J:

$$\mathscr{E} = \rho J. \tag{45}$$

Its reciprocal value is the conductivity, that is, $\sigma = 1/\rho$, and

$$J = \sigma\mathscr{E}. \tag{46}$$

For semiconductors with both electrons and holes as carriers, we obtain

$$\rho = \frac{1}{\sigma} = \frac{1}{q(\mu_n n + \mu_p p)}. \tag{47}$$

If $n \gg p$, as in n-type semiconductors,

$$\rho \simeq \frac{1}{q\mu_n n} \tag{48}$$

or

$$\sigma \simeq q\mu_n n. \tag{48a}$$

The most common method for measuring resistivity is the four-point probe method (insert, Fig. 20).[38,39] A small current from a constant-current source is passed through the outer two probes and the voltage is measured between the inner two probes. For a thin wafer with thickness W much smaller than either a or d, the sheet resistance R_s is given by

$$R_s = \frac{V}{I}\cdot \text{CF} \qquad \Omega/\text{square} \tag{49}$$

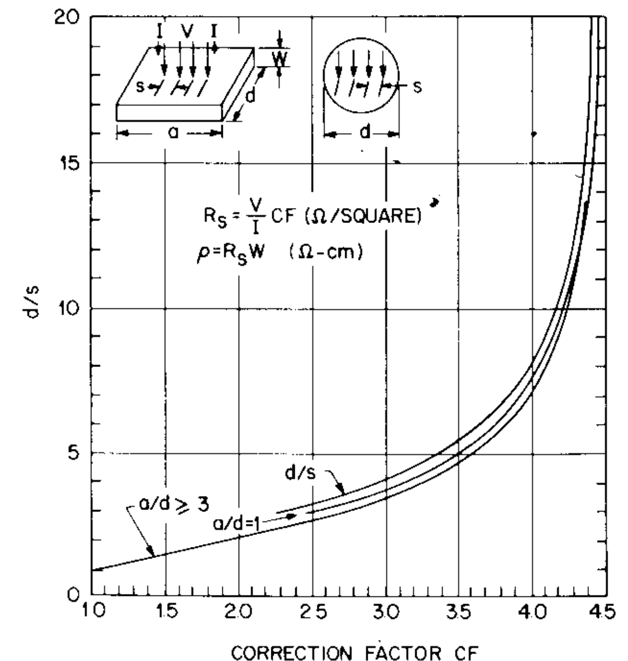

Fig. 20 Correction factor for measurement of resistivity using a four-point probe. (After Beadle, Plummer, and Tsai, Ref. 38.)

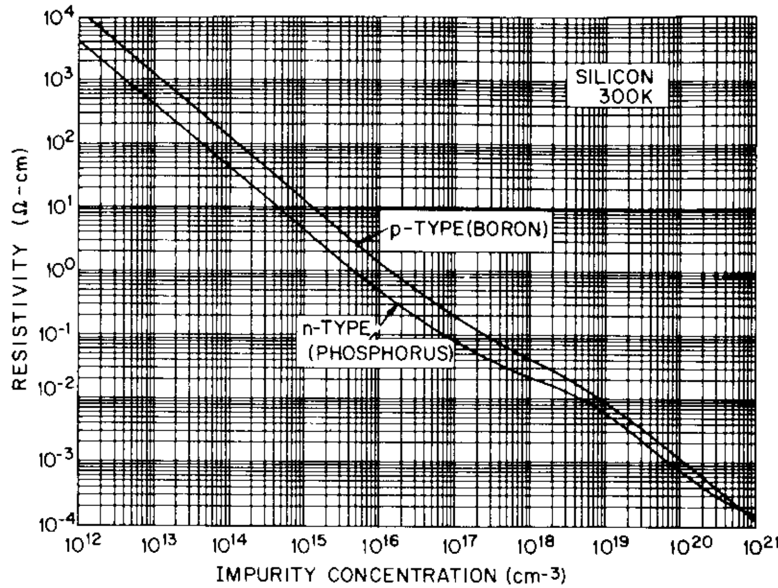

Fig. 21 Resistivity versus impurity concentration for silicon at 300 K. (After Beadle, Plummer, and Tsai, Ref. 38.)

where CF is the correction factor shown in Fig. 20. The resistivity is then

$$\rho = R_s W \qquad \Omega\text{-cm}. \tag{49a}$$

In the limit when $d \gg S$, where S is the probe spacing, the correction factor becomes $(\pi/\ln 2) = 4.54$.

Figure 21 shows the measured resistivity (at 300 K) as a function of the impurity concentration (n-type phosphorus and p-type boron) for silicon.[38] Figure 22 shows the measured resistivities for Ge, GaAs, and GaP.[28,38,40] Thus we can obtain the impurity concentration of a semiconductor if its resistivity is known. The impurity concentration may be different from the carrier concentration. For example, in a p-type silicon with $10^{17}\,\text{cm}^{-3}$ gallium acceptor impurities, un-ionized acceptors at room temperature make up about 23% (from Eq. 25, Figs. 13 and 17); in other words, the carrier concentration is only $7.7 \times 10^{16}\,\text{cm}^{-3}$.

To measure the carrier concentration directly, the most common method uses the Hall effect.[41] Figure 23 shows the basic setup[42] where an electric field is applied along the x axis and a magnetic field is applied along the z axis. Consider a p-type sample. The Lorentz force $q v_x \times \mathscr{B}_z$ exerts an average downward force on the holes, and the downward-directed current causes a piling up of holes at the bottom side of the sample, which in turn gives rise to an electric field $\mathscr{E}_y$. Since there is no net current along the y

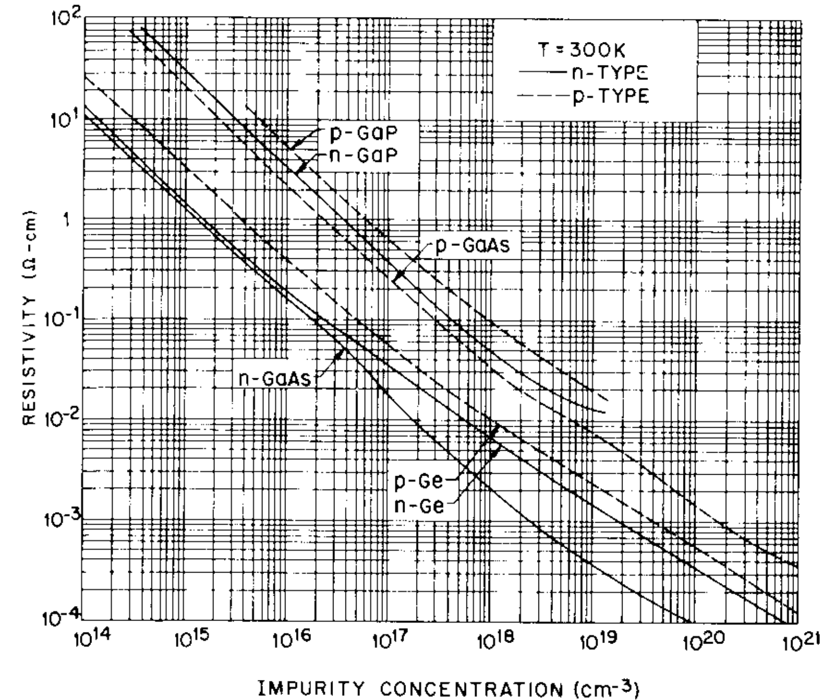

Fig. 22 Resistivity versus impurity concentration for Ge, GaAs, and GaP at 300 K. (After Sze and Irvin, Ref. 28; Beadle, Plummer, and Tsai, Ref. 38.)

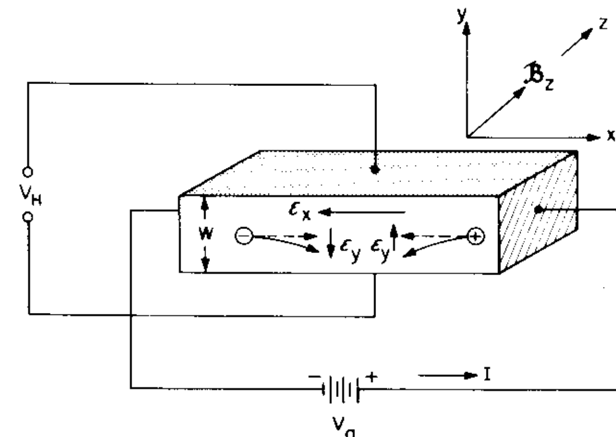

Fig. 23 Basic setup to measure carrier concentration using the Hall effect.