Chemistry 140 a
Lecture 11

Surface, Bulk, and Depletion Region Recombination

Quasi Fermi Levels

For calculations, it would be convenient to assume flat QFLs within a certain region, Δx, under study. Then, the driving force for recombination would be equal everywhere.

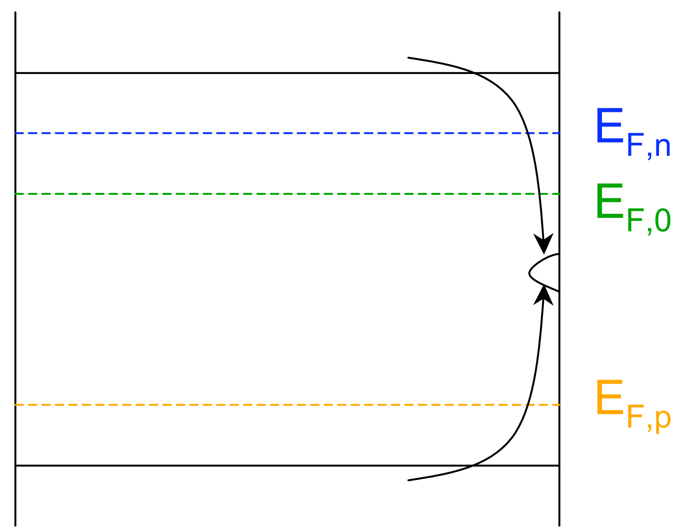
When are QFLs flat?

When Δn or Δp is constant within Δx.

Flat QFLs

QFLs are flat in the bulk when:

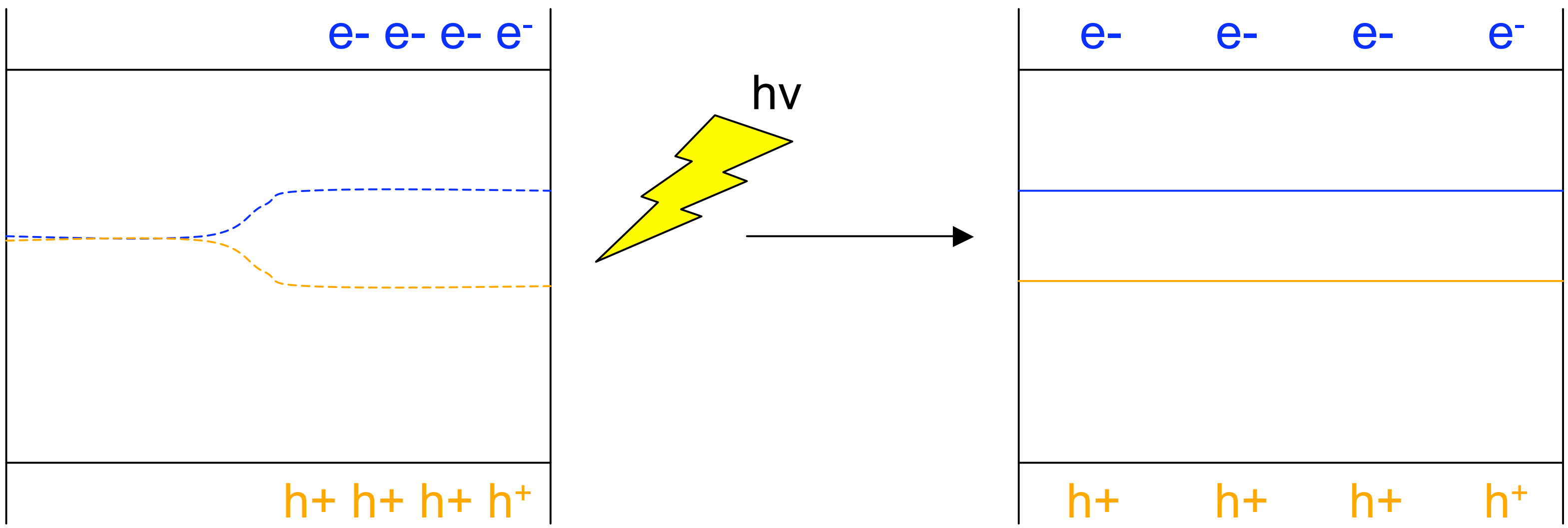
- Recombination at any position x is slow compared to thermal diffusion



Flat QFLs

QFLs are flat in the bulk when:

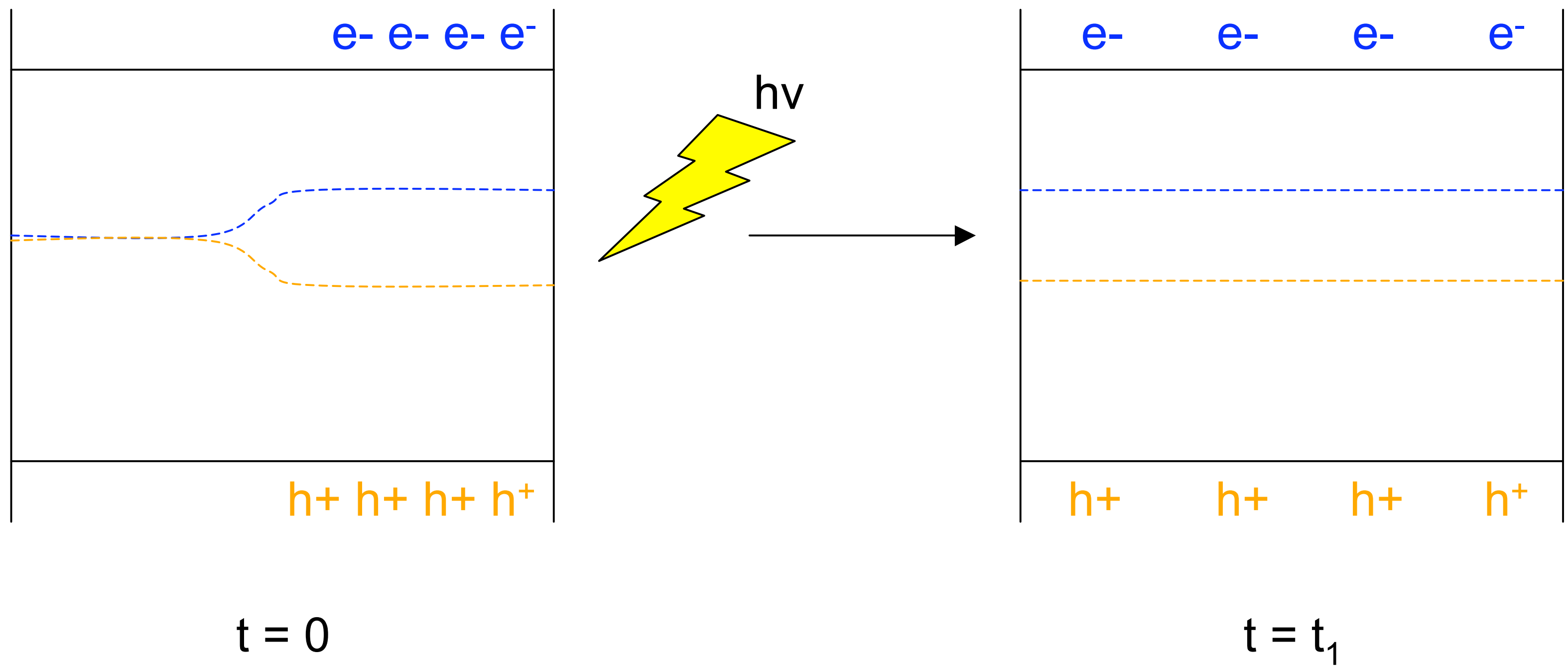
- Light excitation is uniform or light excitation is not uniform but diffusion of carriers flattens QFLs



t = 0
non-uniform QFLs

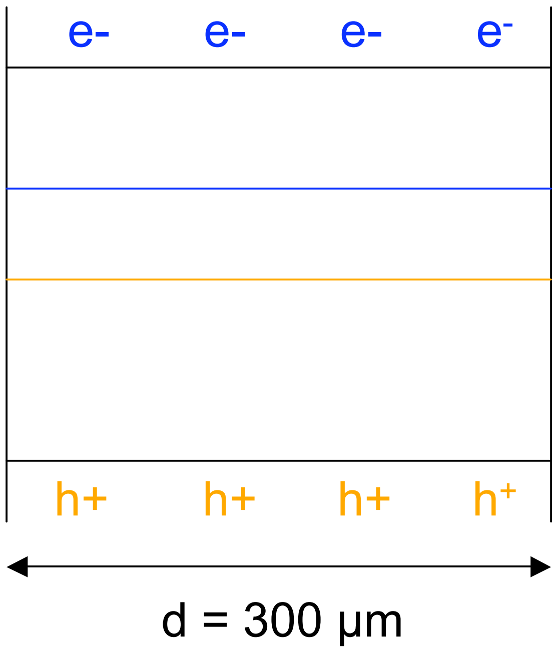
$t = t_1$

Flat QFLs



If t_1, the time it takes for a uniform distribution of carriers to occur via diffusion, is less than τ_{br}, τ_{sr}, or τ_{dr}, then the QFLs are flat.

Diffusion Time in Si



$$L = \sqrt{\frac{D}{\hat{o}}}$$

$$\hat{o} = \frac{L^2}{D}$$

$$L_{eff} = \frac{thickness}{2}$$

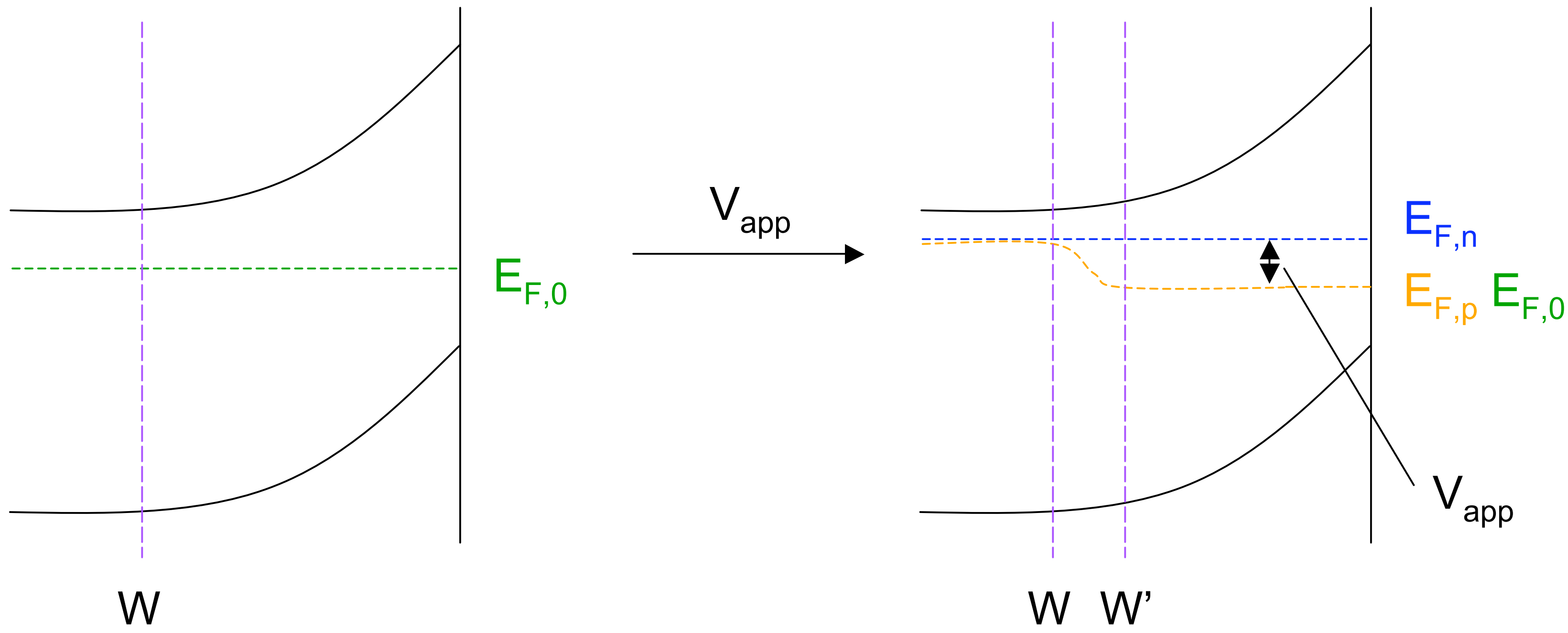
$$D = \frac{D_n D_p}{D_n + D_p}$$

$$\hat{o} = \frac{(0.015\ cm)^2}{8.93\ cm^2 s^{-1}} \approx 25\ \text{ìì}$$

For recombination greater than about 25 μs, we can assume flat QFLs.

Depletion Region Recombination

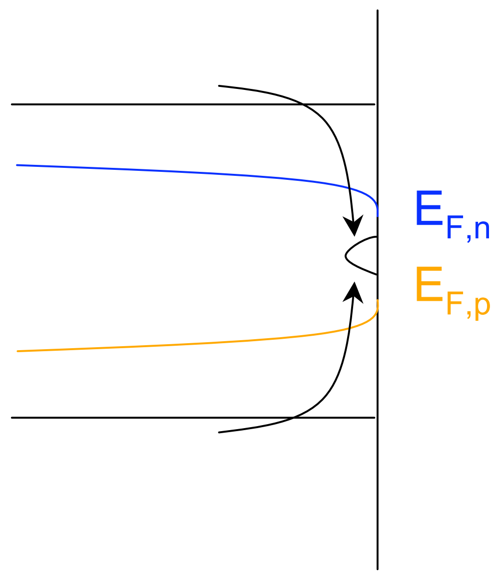
Let's examine the depletion region after applying a bias V_{app}:



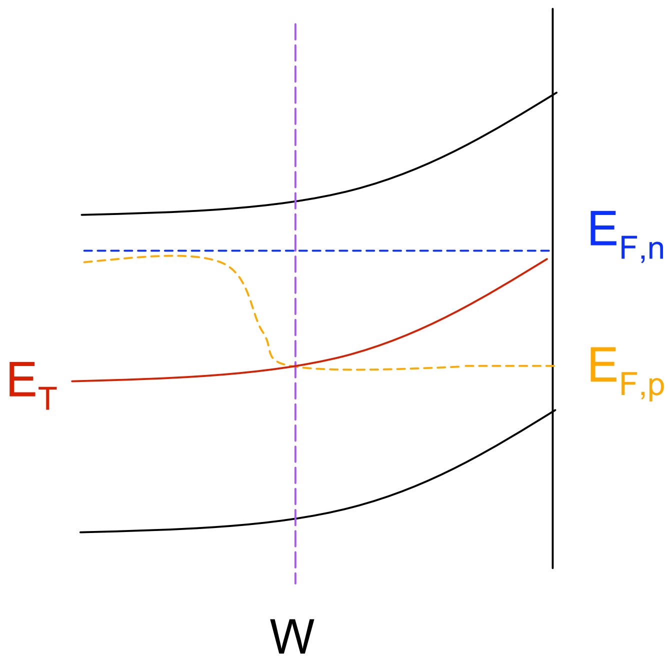
The new effective depletion region is W' < x < 0.
W to W' is a quasi-neutral region, and there is no field there.
It is like the bulk, except $E_{F,p}$ changes with x.
Quasi Fermi levels are flat within this new depletion region.

QFLs Not Flat

For flat QFLs in the depletion region, recombination at the surface must be slow relative to diffusion of carriers. If surface recombination is fast relative to diffusion of carriers, QFLs will not be flat:



n(x) and p(x) Vary with x



The recombination rate in the depletion region is not like in the bulk or on the surface.

We now need to plug in n(x) and p(x) and integrate over 0 to W.

Depletion Region Recombination

Most general form:

$$U(x) = N_T(x) \frac{k_n(x)\, k_p(x)\, (n(x)p(x) - n_i^2)}{k_n(x)\, (n(x) + n_1) + k_p(x)\, (p(x) + p_1)}$$

Before, we just plugged in $n_b = n(x)$ or $n_s = n(x)$ and $p_b = p(x)$ or $p_s = p(x)$.

Now n(x) and p(x) change from 0 to W because of band bending.

We have to integrate over all n(x) in $0 < x < W$:

$$U_{total} = \int_0^W U(x) dx$$

Depletion Region Recombination

Assumptions:

$N_T(x) = N_T$

n_1 and p_1 are constants with respect to x and do not change with V_{bi} for a given trap:

$$n_1 = N_C \exp(-(E_C - E_T)/kT)$$
$$p_1 = N_V \exp(-(E_T - E_V)/kT)$$

k_n and k_p are constant (this can fail since $\sigma(E_T)$ may vary due to ionized or unionized trap states, e.g. $Zn^{2+} \rightarrow Zn^{+}$):

$$n(x) = N_C \exp[-(E_C(x) - E_{F,n})/kT]$$
$$p(x) = N_V \exp[-(E_{F,p} - E_V(x))/kT]$$

$n(x)p(x)$ Not Dependent on x

If $E_{F,n}$ and $E_{F,p}$ are constant with x, then $n(x)p(x)$ is a constant with x.

$$n(x)p(x) = N_C N_V \exp[-(E_C(x)-E_{F,n}+E_{F,p}-E_V(x))/kT]$$
$$= N_C N_V \exp[-(E_C(x)-E_V(x))/kT] \exp[-(E_{F,p}-E_{F,n})/kT]$$

$$E_C(x)-E_V(x) = E_g(x) = E_g \text{ everywhere}$$

$$n(x)p(x) = N_C N_V \exp[-E_g/kT] \exp[-(E_{F,p}-E_{F,n})/kT]$$
$$n(x)p(x) = n_i^2 \exp[-(E_{F,p}-E_{F,n})/kT]$$

$$-(E_{F,p}-E_{F,n}) = qV_{app}$$

$$n(x)p(x) = n_i^2 \exp[-qV_{app}/kT]$$

No dependence on x. Increases in $-V_{app}$ result in $n(x)p(x) > n_i^2$.

Depletion Region Recombination

Returning to U(x)…

$$U(x) = N_T \frac{k_n k_p (n_i^2 \exp[-qV_{app}/kT] - n_i^2)}{k_n(n(x)+n_1) + k_p(p(x)+p_1)}$$

$$U(x) = N_T \frac{k_n k_p n_i^2 (\exp[-qV_{app}/kT] - 1)}{k_n(n(x)+n_1) + k_p(p(x)+p_1)}$$

We may ignore the “1” when V_{app} > 0.75 V (V_{app} > 3kT/q).

A harder assumption is to ignore n_1 in the denominator for significant band bending. We must have either high-level injection or large V_{app}.

Determination of U_{total}

Our assumptions:

p_1 and n_1 are negligible

$k_n = k_p = \sigma v$

$$U(x) = (N_T \sigma v) \frac{n_i^2[\exp(-qV_{app}/kT) - 1]}{n(x) + p(x)}$$

$$U_{total} = \int_0^W U(x)dx = \int_0^W (N_T \sigma v) \frac{n_i^2[\exp(-qV_{app}/kT) - 1]}{n(x) + p(x)} dx$$

Determination of U_{max}

There will be some U_{max} in this region (the depletion region) where n(x) = p(x). This is where the denominator is a minimum.

$$U_{\max} = U(x)\Big|_{\text{at x where n(x)=p(x)}} = \frac{N_T\sigma v[n_i^2 \exp(-qV_{app}/kT) - n_i^2]}{n(x)+p(x)}$$

$$\text{Multiply top and bottom by } \frac{1}{\sqrt{\text{n(x)p(x)}}}$$

$$U_{\max} = \frac{N_T\sigma v\left[\frac{n_i^2 \exp(-qV_{app}/kT)}{\sqrt{\text{n(x)p(x)}}} - \frac{n_i^2}{\sqrt{\text{n(x)p(x)}}}\right]}{\sqrt{\frac{\text{n(x)}}{\text{p(x)}}} + \sqrt{\frac{\text{n(x)}}{\text{p(x)}}}}$$

$$\text{At } U_{\max},\ n(x) = p(x) \text{ and } n(x)p(x) = n_i^2 \exp(-qV_{app}/kT)$$

Determination of U_{max}

$$U_{max} = \frac{N_T \sigma v \left[\sqrt{n(x)p(x)} - \frac{n_i^2}{\sqrt{n_i^2 \exp(-qV_{app}/kT)}} \right]}{2}$$

$$U_{max} = \frac{N_T \sigma v}{2} [n_i \exp(-qV_{app}/2kT) - n_i \exp(qV_{app}/2kT)]$$

$$\text{Remember } \sinh(x) = \frac{1}{2}(e^x - e^{-x})$$

If you include the $-n_i^2$ term from $(n(x)p(x)-n_i^2)$, then:

$$U_{max} = N_T \sigma v n_i \sinh(-qV_{app}/2kT)$$

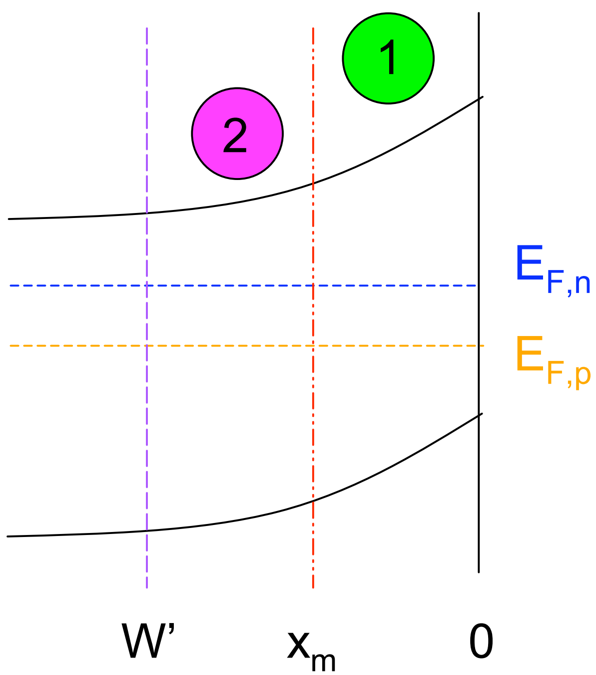
If you exclude the $-n_i^2$ term, then:

$$U_{max} = \frac{N_T \sigma v n_i \exp(-qV_{app}/2kT)}{2}$$

Analysis for x < x_{max} and x > x_{max}

$$U(x) = \frac{N_T \text{óí} \sqrt{n(x)p(x)}}{\sqrt{\frac{n(x)}{p(x)}} + \sqrt{\frac{n(x)}{p(x)}}}$$

$$n(x) = n_{U_{max}} \exp[q\varepsilon_{U_{max}}(x - x_{U_{max}})/kT]$$



1

x < x_m

exp(…) is negative

n(x) < n(x_m)

extra band bending

2

x > x_m

exp(…) is positive

n(x) > n(x_m)

V = ε_mx
= (V/cm)*cm

U(x) in Terms of U_{max}

$$U(x) = \frac{N_T \text{óí} \sqrt{n(x)p(x)}}{\sqrt{\frac{n(x)}{p(x)}} + \sqrt{\frac{n(x)}{p(x)}}}$$

$$U(x) = \frac{N_T \text{óí} n_i \exp(-qV_{app}/2kT)}{\sqrt{\frac{n_m \exp[q\varepsilon_m (x-x_m)/kT]}{p_m \exp[-q\varepsilon_m (x-x_m)/kT]}} + \sqrt{\frac{p_m \exp[-q\varepsilon_m (x-x_m)/kT]}{n_m \exp[q\varepsilon_m (x-x_m)/kT]}}}$$

Since $U_{\max} = \left[\tfrac{1}{2} N_T \sigma v\right] n_i \exp(-qV_{app}/2kT)$...

$$U(x) = \frac{N_T \text{óí} \dfrac{U_{\max}}{\frac{1}{2} N_T \text{óí}}}{\left(\dfrac{n_m}{p_m}\exp[2q\varepsilon_m (x-x_m)/kT]\right)^{1/2} + \left(\dfrac{p_m}{n_m}\exp[-2q\varepsilon_m (x-x_m)/kT]\right)^{1/2}}$$

Since $n_m = p_m$...

$$U(x) = \frac{U_{\max}}{\frac{1}{2}(\exp[q\varepsilon_m (x-x_m)/kT] + \exp[-q\varepsilon_m (x-x_m)/kT])}$$

Remember that $\cosh(x) = \frac{1}{2}[\exp(x) + \exp(-x)]$...

$$U(x) = \frac{U_{\max}}{\cosh[q\varepsilon_m (x-x_m)/kT]}$$

U_{total} in Terms of U_{max}

$$U_{total} = \int_0^W U(x)dx = \int_0^W \frac{U_{\max} dx}{\cosh[\frac{q}{kT}\varepsilon(x - x_m)]}$$

For normally doped semiconductors, the maximum is strongly peaked away from W, so extend the integral to ∞.

$$U_{total} = \int_0^\infty \frac{U_{\max} dx}{\cosh[\frac{q}{kT}\varepsilon(x - x_m)]}$$

$$\int_0^\infty \frac{dx'}{\cosh(x')} = \frac{\pi}{2}$$

$$x' = \tfrac{q}{kT}\varepsilon_m (x - x_m) \text{ and } dx' = \tfrac{q}{kT}\varepsilon_m dx$$

$$U_{total} = \int_0^\infty \frac{U_{\max}}{\frac{q\varepsilon_m}{kT}} \frac{dx'}{\cosh(x')}$$

$$U_{total} = \frac{\pi}{2}\frac{kT}{q\varepsilon_m} U_{\max}$$

U_{total}

Replace exp(…) with sinh(…) if you want to include the $-n_i^2$ term we neglected.

$$\varepsilon_m = (V_{bi} + V_{app}) / W$$

$$U_{total} = \frac{\pi kTW}{2q(V_{bi} + V_{app})} U_{\max}$$

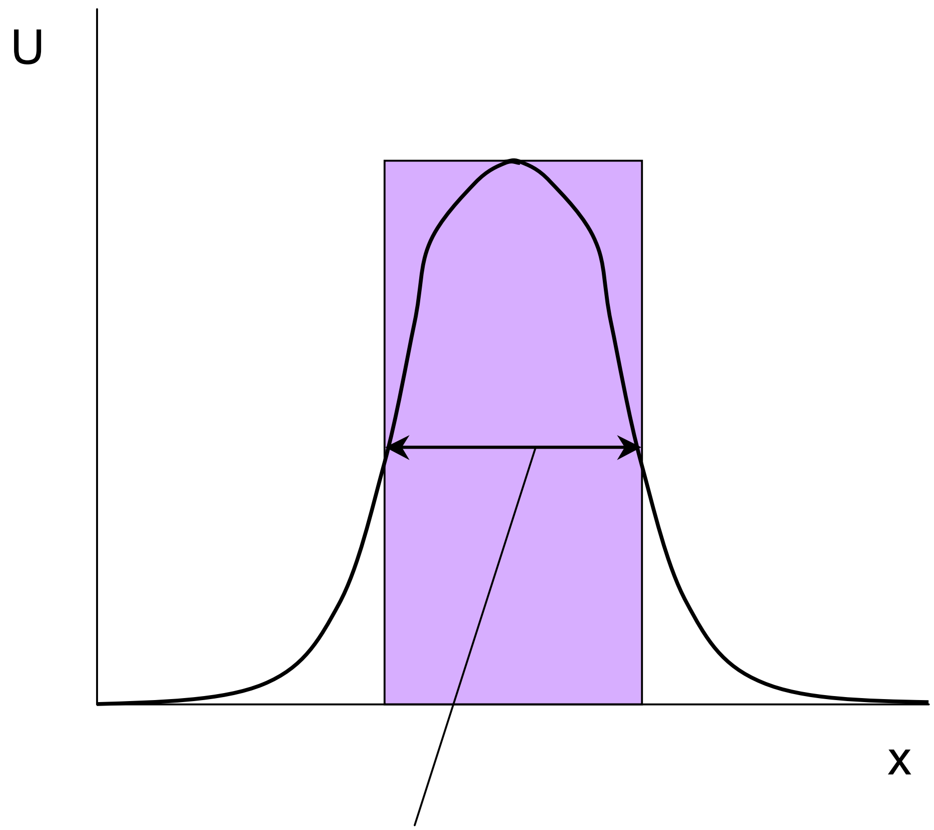
$$U_{total} = \frac{\pi kTW (N_T \sigma v) n_i \exp(-qV_{app} / 2kT)}{4q(V_{bi} + V_{app})}$$

$$J = qU_{total} = \frac{\pi}{2} \frac{kTW}{2(V_{bi} + V_{app})} N_T \sigma v n_i \exp(-qV_{app} / 2kT)$$

Important term. Not like thermionic emission, where it went like exp(-qV_{app}/kT).

The e^- and h^+ recombination is as if we lost half of the voltage to the other carrier.

U vs. x



$$W\frac{kT}{q}\frac{1}{V_{bi}+V_{app}}$$

…a dimensionless quantity that is the ratio of the thermal to applied voltage.

End

Surface, Bulk, and Depletion Region Recombination