

Chem 140a

# Lecture Notes #9: Class #11

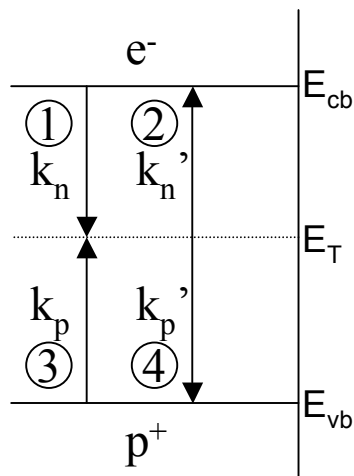
**Recombination under different light intensity**  
**- Low level injection vs High level injection**

<Review>

We looked at recombination/generation

: We usually work away from the equilibrium. so we want to understand R-G process

Typically we add more carriers so that  $np > n_i^2 \rightarrow$  Recombination is dominant



$$r_n = \left. \frac{dn}{dt} \right|_R = \left. \frac{dn}{dt} \right|_1 + \left. \frac{dn}{dt} \right|_2 = -k_n \cdot n_b \cdot (1 - f_T) \cdot N_T + k_n' \cdot f_T \cdot N_T$$

$$r_p = \left. \frac{dp}{dt} \right|_R = \left. \frac{dp}{dt} \right|_3 + \left. \frac{dp}{dt} \right|_4 = -k_p \cdot p_b \cdot f_T \cdot N_T + k_p' \cdot (1 - f_T) \cdot N_T$$

By applying equilibrium conditions: Principle of detailed balance

$$\boxed{r_n = r_p = 0}$$

$$k_n n_{b,0} (1 - f_T) N_T = k_n' f_T N_T$$

$$k_n' = \frac{1 - f_T}{f_T} k_n n_{b,0}$$

<Review>

Then we calculated

$$\frac{1-f_T}{f_T} = \exp[-(E_F - E_T)/kT]$$

$$\therefore k_n' = k_n \left( \frac{1-f_T}{f_T} \right) n_{b,0} = k_n n_{b,0} \exp[-(E_F - E_T)/kT] = k_n N_C \exp[-(E_C - E_T)/kT]$$

$$(\because n_{b,0} = N_C \exp[-(E_C - E_F)/kT])$$

$k_n'$  doesn't depend on the Fermi level and only depends on the trap energy level

$$\text{Let } n_1 = N_C \exp[-(E_C - E_T)/kT]$$

$$p_1 = N_V \exp[-(E_T - E_V)/kT] \text{ Then,}$$

$$\underline{k_n' = k_n n_1}$$

$$\underline{k_p' = k_p p_1}$$

By plugging in  $k_n'$  and  $k_p'$ ,

$$r_n = \left. \frac{dn}{dt} \right|_R = -k_n \cdot n_b \cdot (1-f_T) \cdot N_T + k_n \cdot n_1 \cdot f_T \cdot N_T$$

$$r_p = \left. \frac{dp}{dt} \right|_R = -k_p \cdot p_b \cdot f_T \cdot N_T + k_p \cdot p_1 \cdot (1-f_T) \cdot N_T$$

<Review>

For steady state assumed case,

$$U_{bulk} = -\left. \frac{dn}{dt} \right|_R = -\left. \frac{dp}{dt} \right|_R = \text{recombination rate}$$

Plug in our values for  $r_n$  &  $r_p$

$$k_n \cdot n_b \cdot (1 - f_T) \cdot N_T - k_n \cdot n_1 \cdot f_T \cdot N_T = k_p \cdot p_b \cdot f_T \cdot N_T - k_p \cdot p_1 \cdot (1 - f_T) \cdot N_T$$

$N_T$ s cancel and solve for  $f_T$

$$f_T = \frac{k_n \cdot n_b + k_p \cdot p_1}{k_n \cdot n_b + k_n \cdot n_1 + k_p \cdot p_b + k_p \cdot p_1}$$

$$1 - f_T = \frac{k_p \cdot p_b + k_n \cdot n_1}{k_n \cdot n_b + k_n \cdot n_1 + k_p \cdot p_b + k_p \cdot p_1}$$

Now that we have  $f_T$  and  $1 - f_T$  we can use them to calculate U.

<Review>

$$\begin{aligned}U_{bulk} &= -\left. \frac{dn}{dt} \right|_R = -\left. \frac{dp}{dt} \right|_R \\&= -k_n \cdot n_b \cdot (1 - f_T) \cdot N_T + k_n \cdot n_1 \cdot f_T \cdot N_T \\U_{bulk} &= N_T \frac{k_n \cdot n_b \cdot (k_p \cdot p_b + k_n \cdot n_1) - k_n \cdot n_1 \cdot (k_n \cdot n_b + k_p \cdot p_1)}{k_n \cdot (n_b + n_1) + k_p \cdot (p_b + p_1)} \\U_{bulk} &= N_T \frac{k_n \cdot k_p \cdot (n_b \cdot p_b - n_1 \cdot p_1)}{k_n \cdot (n_b + n_1) + k_p \cdot (p_b + p_1)} \\n_1 \cdot p_1 &= N_C \exp[-(E_C - E_T)/kT] \cdot N_V \exp[-(E_T - E_V)/kT] \\&= N_C \cdot N_V \cdot \exp[-(E_C - E_V)/kT] = n_i^2\end{aligned}$$

So the recombination rate in bulk,

$$U_{bulk} = N_T \frac{k_n \cdot k_p \cdot (n_b \cdot p_b - n_i^2)}{k_n \cdot (n_b + n_1) + k_p \cdot (p_b + p_1)}$$

## Surface Recombination

It turns out we can write the same equation for the surface, but by modifying all terms to be surface related ones

$$U_{\text{surface}} = N_{T,S} \frac{k_{n,s} \cdot k_{p,s} \cdot (n_s \cdot p_s - n_i^2)}{k_{n,s} \cdot (n_s + n_{1,s}) + k_{p,s} \cdot (p_s + p_{1,s})}$$

$N_{T,S}$  : different from the bulk. Unit is  $[\text{cm}^{-2}]$ , not  $[\text{cm}^{-3}]$

→  $k_{n,s}$  and  $k_{p,s}$  are different from the bulk

$E_{T,S}$  is different from  $E_T$  (bulk) →  $n_{1,s}$  is different from  $n_1$  (bulk)

$n_i^2$  is the only term that is the same

# Surface vs Bulk Recombination

## Units

$$U_{bulk} = N_T \frac{k_n \cdot k_p \cdot (n_b \cdot p_b - n_i^2)}{k_n \cdot (n_b + n_1) + k_p \cdot (p_b + p_1)}$$

$$\rightarrow U_{bulk} : \frac{\text{particles}}{cm^3 \cdot s} = [cm^{-3}] \frac{\left[\frac{cm^3}{s}\right] \left[\frac{cm^3}{s}\right] [cm^{-6}]}{\left[\frac{cm^3}{s}\right] [cm^{-3}]}$$

$$U_{surface} = N_{T,S} \frac{k_{n,s} \cdot k_{p,s} \cdot (n_s \cdot p_s - n_i^2)}{k_{n,s} \cdot (n_s + n_{1,s}) + k_{p,s} \cdot (p_s + p_{1,s})}$$

$$\rightarrow U_{surface} : \frac{\text{particles}}{cm^2 \cdot s} = \text{flux} = [cm^{-2}] \frac{\left[\frac{cm^3}{s}\right] \left[\frac{cm^3}{s}\right] [cm^{-6}]}{\left[\frac{cm^3}{s}\right] [cm^{-3}]}$$

## Surface vs Bulk Recombination

For the bulk recombination,

$$U_{bulk} = N_T \frac{k_n \cdot k_p \cdot (n_b \cdot p_b - n_i^2)}{k_n \cdot (n_b + n_1) + k_p \cdot (p_b + p_1)}$$

Divide by  $k_n \cdot k_p \cdot N_T$

$$\rightarrow U_{bulk} = \frac{n_b \cdot p_b - n_i^2}{\frac{1}{N_T \cdot k_p} \cdot (n_b + n_1) + \frac{1}{N_T \cdot k_n} \cdot (p_b + p_1)}$$

$$N_T \cdot k_p : [cm^{-3}] \cdot [cm^3 / s] = 1 / s \rightarrow \frac{1}{N_T \cdot k_p} = \tau_p$$

Similarly,

$$\frac{1}{N_T \cdot k_n} = \tau_n$$

where  $\tau$  is the lifetime of carrier

$$U_{bulk} = \frac{n_b \cdot p_b - n_i^2}{\tau_p \cdot (n_b + n_1) + \tau_n \cdot (p_b + p_1)}$$



## Surface vs Bulk Recombination

For the bulk recombination,

$$\tau_n = \frac{1}{N_T \cdot k_n} \quad / \quad \tau_p = \frac{1}{N_T \cdot k_p}$$

What is a good values for  $\tau_n$  or  $\tau_p$  ?

Recall  $k_n = \text{cm}^3 / \text{s} = \text{cm}^2 \cdot \text{cm} / \text{s} = \sigma V_{th}$

Assume atom radius of 3 Å,  $\pi r^2 = 10^{-15} \text{ cm}^2$

$$V_{th} \approx 10^7 \text{ cm} / \text{s} \quad \rightarrow k_n = 10^{-15} \cdot 10^7 \text{ cm}^3 / \text{s} = 10^{-8} \text{ cm}^3 / \text{s}$$

Good approximation to say " $k_n = k_p$ "

For low trap density in the bulk, let's say  $10^{11} \text{ cm}^{-3}$

$$\rightarrow \tau_{n,b} = \frac{1}{10^{11} \cdot 10^{-8}} = \frac{1}{10^3} = \text{ms} \text{ (About right for bulk Si)}$$

But for the surface recombination, this doesn't work the same way

## Bulk Recombination: under Low Level Injection

For n-type bulk,

Low-level injection

$$N_D = n_{b,0} \gg \Delta n = \Delta p \gg p_{b,0}$$

$$U_{bulk}(LLI) = N_T \frac{k_n \cdot k_p \cdot [(N_D + \Delta n) \cdot (\Delta p + p_{b,0}) - n_i^2]}{k_n \cdot (N_D + n_1) + k_p \cdot (p + p_1)}$$

$\nwarrow N_D \gg n_i \approx n_1 \quad \nwarrow p = \Delta p + p_{b,0}$

$$U_{bulk}(LLI) = N_T \frac{k_n \cdot k_p \cdot [N_D \Delta p + \Delta n \Delta p + N_D p_{b,0} + \Delta n p_{b,0} - n_i^2]}{k_n \cdot (N_D) + k_p \cdot (p + p_1)}$$

$(n_i^2 = n_{b,0} p_{b,0} = N_D p_{b,0} \rightarrow$  So  $N_D p_{b,0}$  and  $n_i^2$  are canceled each other

$$U_{bulk}(LLI) = N_T \frac{k_n \cdot k_p \cdot N_D \Delta p}{k_n \cdot (N_D) + k_p \cdot (\Delta p + p_{b,0} + p_1)}$$

If  $k_n \approx k_p$ , then whole  $k_p$  terms go away.

$$\boxed{U_{bulk}(LLI) = N_T k_p \Delta p}$$

## Bulk Recombination: under Low Level Injection

$$U_{bulk} (LLI) = N_T k_p \Delta p$$

Remember, rate=(rate constant)×(concentration)

We can generalize and define

$$U \equiv S \Delta p \quad \text{for n-type}$$

where S, the recombination velocity is a rate constant

For low-level injection in the bulk,

$$\boxed{S_{bulk} (LLI) = N_T k_p = \frac{1}{\tau_p}} \quad \text{for n-type}$$

$$U = S \Delta p \quad \& \quad U = -\frac{\partial \Delta n}{\partial t} = -\frac{\partial \Delta p}{\partial t}$$
$$-\frac{\partial \Delta p}{\partial t} = S \Delta p \quad \Rightarrow \quad -\frac{\partial \Delta p}{\Delta p} = S \partial t$$

$$\Delta p = p_0 \exp(-St) = p_0 \exp(-t / \tau_p)$$

## Bulk Recombination: under High Level Injection

For n-type bulk,

High-level injection

$$\Delta n = \Delta p \gg n_{b,0} = N_D \gg p_{b,0}$$

$$U_{bulk}(HLI) = N_T \frac{k_n \cdot k_p \cdot [(N_D + \Delta n) \cdot (p_{b,0} + \Delta p) - n_i^2]}{k_n \cdot (N_D + \Delta n + n_1) + k_p \cdot (p_{b,0} + \Delta p + p_1)}$$

$$U_{bulk}(HLI) = N_T \frac{k_n \cdot k_p \cdot \Delta n \cdot \Delta p}{k_n \cdot (\Delta n) + k_p \cdot (\Delta p)}$$

But  $\Delta n = \Delta p$

$$U_{bulk}(HLI) = N_T \frac{k_n \cdot k_p}{k_n + k_p} \Delta p$$

$$\text{Now } S_{bulk}(HLI) = N_T \frac{k_n \cdot k_p}{k_n + k_p} = \frac{1}{\frac{1}{N_T k_p} + \frac{1}{N_T k_n}}$$

$$S_{bulk}(HLI) = \frac{1}{\tau_n + \tau_p}$$

$\Rightarrow$  The carrier with the longer life time determines the recombination velocity

## **Bulk Recombination:** under High Level Injection

Finally, if  $k_n \approx k_p$ , then  $\tau_n \approx \tau_p \equiv \tau_{bulk}$

$$S_{bulk}(HLI) = \frac{1}{\tau_n + \tau_p} = \frac{1}{2\tau_{bulk}} = \frac{1}{2\tau_p}$$

$$S_{bulk}(LLI) = \frac{1}{\tau_p}$$

This results show that  $S_{bulk}(HLI) = \frac{1}{2} S_{bulk}(LLI)$

Under low level injection, we only have to wait for the photogenerated holes,  $\Delta p$  to relax.

This is because  $n = N_D + \Delta n \cong N_D$ .

The perturbed electron concentration is essentially zero.

But for high level injection, we have to wait for both electrons ( $\Delta n$ ) and holes ( $\Delta p$ ) to relax.

This will take twice as long if  $\tau_n \approx \tau_p$ . (or else it will take  $\tau_n + \tau_p$ )