Chem 140a

Lecture Notes #8: Class #10

Recombination-Generation Statistics (R-G)
(Shockley-Read-Hall statistics)

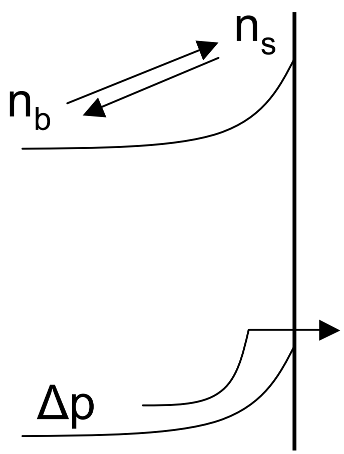
<Review>

Photocurrent: due to minority carrier current generated from photoinduced holes, Δp

$$h\nu \rightarrow \Delta n = \Delta p$$

$$n = N_D \gg \Delta n$$

$$\Delta p \gg p$$



If band bending is steep enough, all Δp cross the interface and J_{ph} is a constant

$$J = -J_0[\exp(-qV/AkT) - 1] + J_{ph}$$

at Voc, $J = 0$ So,

$$J_0[\exp(-qV/AkT) - 1] = J_{ph}$$

if $V > \dfrac{3kT}{q}$, we can ignore "1"

$$J_0[\exp(-qV/AkT)] = J_{ph}$$

$$\therefore |V_{oc}| = \frac{AkT}{q}\ln\frac{J_{ph}}{J_0}$$

⇐ **Name of this game is to minimize J_0 to get large V_{oc}**

<Review>

SC/Metal interface,

$$J_0 = A^* T^2 \exp(-\frac{q\phi_b}{kT})$$ **: we want to have large ϕ_b to minimize J_0**

SC/Solution interface,

$$J_0 = qk_{et}[A]n_{s0}$$

$$J_{0,soln} < J_{0,m}$$

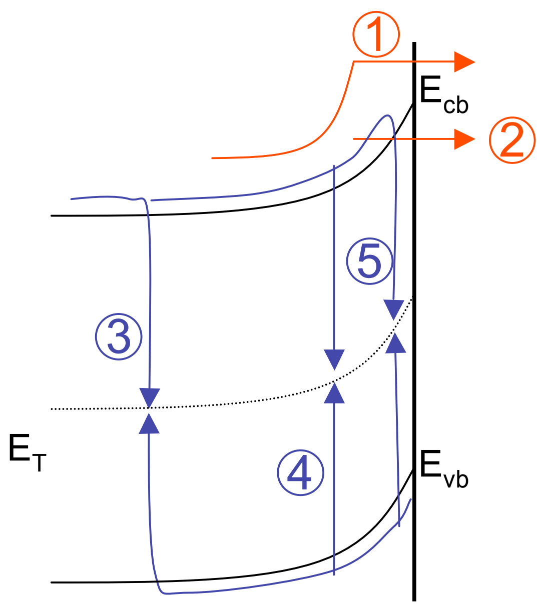
MIS,

insulator induces a less than unity probability for the e^- to corss the SC interface

Limit of ϕ_b,

Max ϕ_b can only be as big as band gap energy, E_g

Five Mechanisms



Majority carrier process

① **Thermionic Emission**

② **Tunneling**

Minority carrier process

③ **Bulk Recombination**

④ **Depletion Region Recombination**

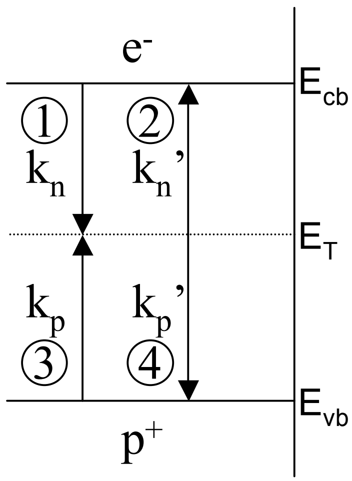
⑤ **Surface Recombination**

Under light or a bias, $np > n_i^2$. So there is a driving force to return to the equilibrium. Extra holes will recombine with extra electrons ➔ Recombination

If somehow we were sucking carriers out of a region of the SC, $np < n_i^2$ and carriers would be generated ➔ Generation

Shockley-Read-Hall statistics: Good model for all of these minority carrier processes of recombination in the SC

Bulk Recombination



For recombination (loss of carriers),

$$\left.\frac{dn}{dt}\right|_R = \left.\frac{dn}{dt}\right|_1 + \left.\frac{dn}{dt}\right|_2$$

$$\left.\frac{dp}{dt}\right|_R = \left.\frac{dp}{dt}\right|_3 + \left.\frac{dp}{dt}\right|_4$$

f_T **= fraction of filled trap states**

$1 - f_T$ **= fraction of empty trap states**

① $$\left.\frac{dn}{dt}\right|_1 = -k_n \cdot n_b \cdot (1 - f_T) \cdot N_T$$

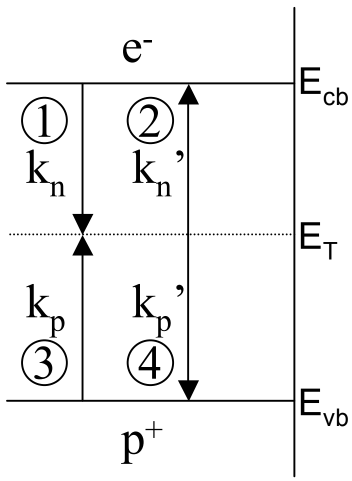
- $\left.\frac{dn}{dt}\right|_1$ → **[cm^{-3}sec^{-1}]**
- k_n → **[cm^3/sec] : has an analogy with** σV_{th}
  where σ **is cross-section and** V_{th} **is velocity**
- n_b → **[cm^{-3}]**
- $(1 - f_T)$ → **(unitless)**
- N_T → **[cm^{-3}]**

② $$\left.\frac{dn}{dt}\right|_2 = k_n' \cdot f_T \cdot N_T$$

③ $$\left.\frac{dp}{dt}\right|_3 = -k_p \cdot p_b \cdot f_T \cdot N_T$$

④ $$\left.\frac{dp}{dt}\right|_4 = k_p' \cdot (1 - f_T) \cdot N_T$$

Bulk Recombination



$$\mathbf{r_n} = \left.\frac{dn}{dt}\right|_R = \left.\frac{dn}{dt}\right|_1 + \left.\frac{dn}{dt}\right|_2 = -k_n \cdot n_b \cdot (1 - f_T) \cdot N_T + k_n' \cdot f_T \cdot N_T$$

$$\mathbf{r_p} = \left.\frac{dp}{dt}\right|_R = \left.\frac{dp}{dt}\right|_3 + \left.\frac{dp}{dt}\right|_4 = -k_p \cdot p_b \cdot f_T \cdot N_T + k_p' \cdot (1 - f_T) \cdot N_T$$

Equilibrium conditions: Principle of detailed balance

➔ All fundamental processes and their inverse processes must self balance

$$\boxed{\mathbf{r_n} = \mathbf{r_p} = 0}$$

electrons,

$$k_n n_{b,0} (1 - f_T) N_T = k_n' f_T N_T$$

$$k_n' = \frac{1 - f_T}{f_T} k_n n_{b,0}$$

Bulk Recombination

For electrons,

$$k_n n_{b,0}(1-f_T)N_T = k_n' f_T N_T$$

$$k_n' = \frac{1-f_T}{f_T} k_n n_{b,0}$$

What is the ratio, $\frac{1-f_T}{f_T}$?

Probability that a trap is filled is given by Fermi-Dirac Statistics.

$$f_T = \frac{1}{\exp[-(E_F - E_T)/kT]+1}$$

$$1-f_T = \frac{1}{\exp[(E_F - E_T)/kT]+1}$$

$$\frac{f_T}{1-f_T} = \frac{\frac{1}{\exp[-(E_F - E_T)/kT]+1}}{1-\frac{1}{\exp[-(E_F - E_T)/kT]+1}} \times \frac{\exp[-(E_F - E_T)/kT]+1}{\exp[-(E_F - E_T)/kT]+1} = \frac{1}{\exp[-(E_F - E_T)/kT]+1-1}$$

$$\boxed{\therefore \frac{f_T}{1-f_T} = \frac{1}{\exp[-(E_F - E_T)/kT]}}$$

Bulk Recombination

$$\frac{1-f_T}{f_T} = \exp[-(E_F - E_T)/kT]$$

$$\therefore k_n' = k_n\left(\frac{1-f_T}{f_T}\right)n_{b,0} = k_n n_{b,0} \exp[-(E_F - E_T)/kT] = k_n N_C \exp[-(E_C - E_T)/kT]$$

$$(\because\ n_{b,0} = \mathrm{N_C} \exp[-(E_C - E_F)/kT])$$

k_n' doesn't depend on the Fermi level and only depends on the trap energy level

For a given trap energy, $\mathrm{N_C} \exp[-(E_C - E_T)/kT]$ is a constant.

Let $\mathrm{n_1} = \mathrm{N_C} \exp[-(E_C - E_T)/kT]$

Then, $\underline{k_n' = k_n n_1}$

Similarly, $\underline{k_p' = k_p p_1}$ where $\mathrm{p_1} = \mathrm{N_V} \exp[-(E_T - E_V)/kT]$

By plugging in k_n' and k_p' ,

$$\mathrm{r_n} = \left.\frac{dn}{dt}\right|_R = -k_n \cdot n_b \cdot (1 - f_T) \cdot N_T + k_n \cdot n_1 \cdot f_T \cdot N_T$$

$$\mathrm{r_p} = \left.\frac{dp}{dt}\right|_R = -k_p \cdot p_b \cdot f_T \cdot N_T + k_p \cdot p_1 \cdot (1 - f_T) \cdot N_T$$

Bulk Recombination

Steady state relationship: Major assumption in device problems and analysis

At steady state, $\mathbf{r_n = r_p}$
(Different from the equilibrium)

$$U_{bulk} = -\left.\frac{dn}{dt}\right|_R = -\left.\frac{dp}{dt}\right|_R = \text{ recombination rate}$$

Plug in our values for r_n & r_p

$$k_n \cdot n_b \cdot (1 - f_T) \cdot N_T - k_n \cdot n_1 \cdot f_T \cdot N_T = k_p \cdot p_b \cdot f_T \cdot N_T - k_p \cdot p_1 \cdot (1 - f_T) \cdot N_T$$

N_Ts cancel and solve for f_T

$$\rightarrow k_n \cdot n_b \cdot (1 - f_T) - k_n \cdot n_1 \cdot f_T = k_p \cdot p_b \cdot f_T - k_p \cdot p_1 \cdot (1 - f_T)$$

$$\rightarrow k_n \cdot n_b - k_n \cdot n_b \cdot f_T - k_n \cdot n_1 \cdot f_T = k_p \cdot p_b \cdot f_T - k_p \cdot p_1 + k_p \cdot p_1 \cdot f_T$$

$$f_T = \frac{k_n \cdot n_b + k_p \cdot p_1}{k_n \cdot n_b + k_n \cdot n_1 + k_p \cdot p_b + k_p \cdot p_1}$$

$$1 - f_T = \frac{k_p \cdot p_b + k_n \cdot n_1}{k_n \cdot n_b + k_n \cdot n_1 + k_p \cdot p_b + k_p \cdot p_1}$$

Now that we have f_T and 1-f_T we can use them to calculate U_{bulk}.

Bulk Recombination

$$U_{bulk} = -\frac{dn}{dt}\bigg|_R = -\frac{dp}{dt}\bigg|_R$$

$$= -k_n \cdot n_b \cdot (1 - f_T) \cdot N_T + k_n \cdot n_1 \cdot f_T \cdot N_T$$

$$U_{bulk} = N_T \frac{k_n \cdot n_b \cdot (k_p \cdot p_b + k_n \cdot n_1) - k_n \cdot n_1 \cdot (k_n \cdot n_b + k_p \cdot p_1)}{k_n \cdot (n_b + n_1) + k_p \cdot (p_b + p_1)}$$

$$U_{bulk} = N_T \frac{k_n \cdot k_p \cdot (n_b \cdot p_b - n_1 \cdot p_1)}{k_n \cdot (n_b + n_1) + k_p \cdot (p_b + p_1)}$$

$$n_1 \cdot p_1 = \mathrm{N_C} \exp[-(E_C - E_T)/kT] \cdot \mathrm{N_V} \exp[-(E_T - E_V)/kT]$$

$$= \mathrm{N_C} \cdot \mathrm{N_V} \cdot \exp[-(E_C - E_V)/kT] = n_i^2$$

So the recombination rate in bulk,

$$U_{bulk} = N_T \frac{k_n \cdot k_p \cdot (n_b \cdot p_b - n_i^2)}{k_n \cdot (n_b + n_1) + k_p \cdot (p_b + p_1)}$$