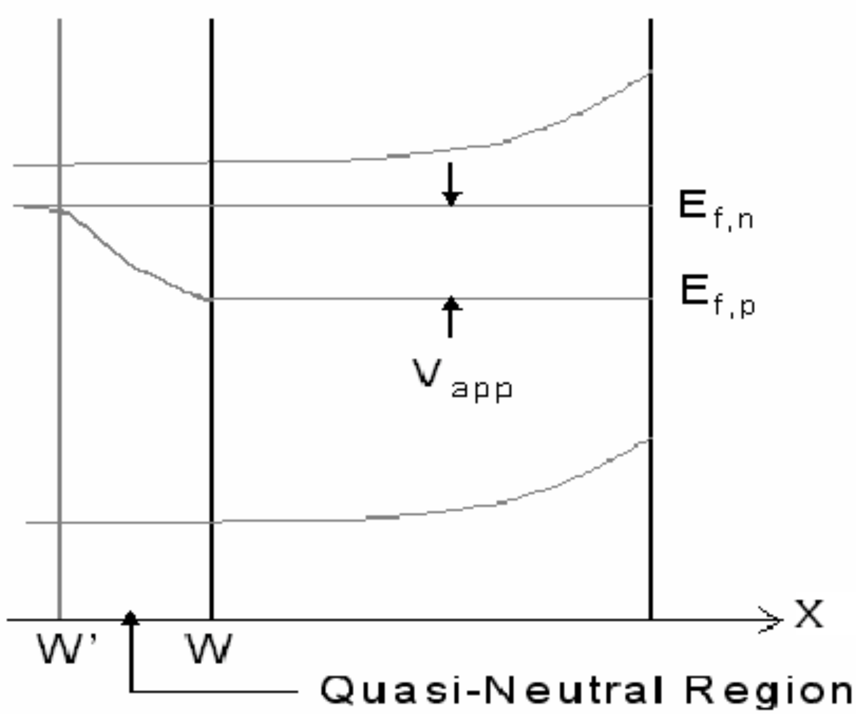
Review: Depletion Region Recombination

Flat Quasi-Fermi Level (QFL)

→ Requires rate limiting recombination is slow compared to thermal diffusion



Note, Vapp < 0 here

Assuming: $k_n \sim k_p = \sigma v$, Flat QFL
→We get for recombination in the bulk at position x,

$$U_{dr}(x) = N_T \sigma v \frac{n(x)p(x) - n_i^2}{n(x)+p(x)}$$

Review: Depletion Region Recombination

$$U_{dr}(x)=N_T \sigma v \frac{n(x)p(x) - n_i^2}{n(x)+p(x)}$$

$$n(x)p(x)=n_i^2 \mathrm{Exp}(-qV_{app}/kT)$$

$$n(x)\,p(x) = \text{constant for flat QFL and constant } V_{app}$$

-Substitute definitions and divide top and bottom by $\sqrt{n(x)p(x)}$

$$U_{dr}(x)=\frac{N_T \sigma v n_i \{\mathrm{Exp}(-qV_{app}/2kT)-\mathrm{Exp}(qV_{app}/2kT)\}}{\sqrt{\frac{n(x)}{p(x)}}+\sqrt{\frac{p(x)}{n(x)}}}$$

Review: Depletion Region Recombination

$$U_{dr}(x)=\frac{2N_T\acute{o}vn_i\ \sinh(-qV_{app}/2kT)}{\sqrt{\frac{n(x)}{p(x)}}+\sqrt{\frac{p(x)}{n(x)}}}$$

Or, for $V_{app}>3kT$

$$U_{dr}(x)=\frac{2N_T\acute{o}vn_i\ Exp(-qV_{app}/2kT)}{\sqrt{\frac{n(x)}{p(x)}}+\sqrt{\frac{p(x)}{n(x)}}}$$

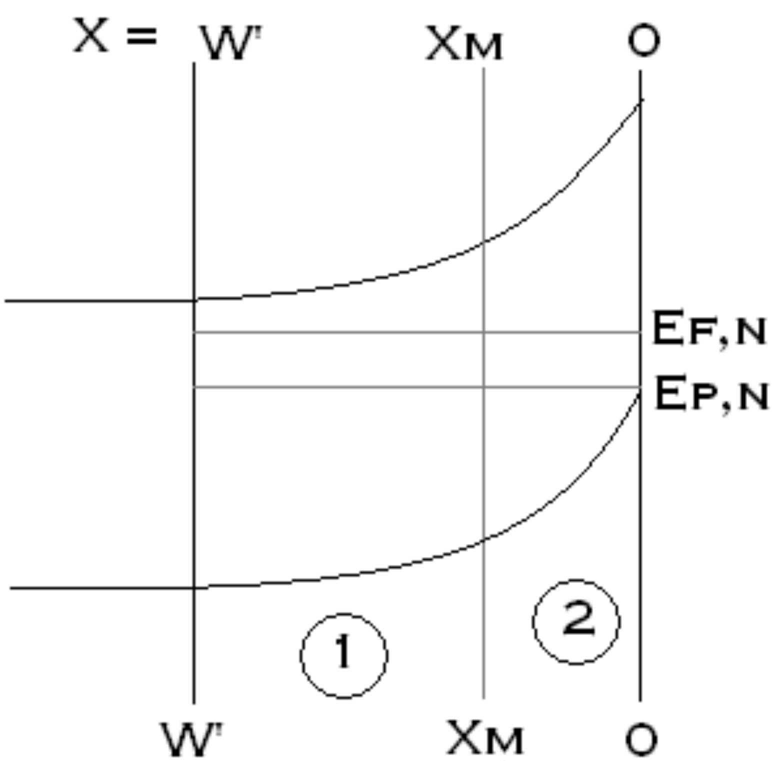
Maximum occurs when n(x) = p(x)

$$U_{dr,max}(x)=\frac{N_T\acute{o}vn_i}{2}\ Exp\left(-\frac{qV_{app}}{2kT}\right)$$

New Material: Recall we desire U_{TOTAL}

Continue with $U_{dr,max}$, recalling we want to solve for U_{TOTAL}, as

$$U_{TOTAL} = \int_0^{W'} U(x)\, dx$$



We need definition for **n(x) and p(x)**

You can show that

$$n(x)=n_m \mathrm{Exp}[q\mathring{a}(x-x_m)/kT]$$

$$p(x)=p_m \mathrm{Exp}[-q\mathring{a}(x-x_m)/kT]$$

$$\mathring{a} = \text{Electric Field}$$

Nm = concentration of e^- for max U,
n(xm)=p(xm)
Xm= position where n=nm

In region 1, x<xm Exp (-) so n(x)<n(xm)
→ More band bending

In region 2, x>xm Exp (+) so n(x)>n(xm)
→ Less band bending

Calculating U_{TOTAL}: Back to Math

Substituting in for n(x) and p(x)

$$U(x)=\frac{2N_T\text{ó}vn_i\text{Exp}(-qV_{app}/2kT)}{\sqrt{\dfrac{n_m\text{Exp}[q\text{å}(x-x_m)/kT]}{p_m\text{Exp}[-q\text{å}(x-x_m)/kT]}}+\sqrt{\dfrac{p_m\text{Exp}[-q\text{å}(x-x_m)/kT]}{n_m\text{Exp}[q\text{å}(x-x_m)/kT]}}}$$

Use $\dfrac{\text{Exp}(x)}{\text{Exp}(-x)}=\text{Exp}(2x)$ and $n_m=p_m$

$$U(x)=\frac{N_T\text{ó}vn_i\text{Exp}(-qV_{app}/2kT)}{\text{Exp}[q\text{å}(x-x_m)/kT]+\text{Exp}[-q\text{å}(x-x_m)/kT]}$$

Calculating U_{TOTAL}

Use definition of $\cosh x = \frac{e^{x}+e^{x}}{2}$, and formula for U_{MAX}

$$U(x) = \frac{U_{MAX}}{\cosh[q\text{å}(x-x_m)/kT]}$$

$$U_{TOTAL} = \int_0^{W'} U(x)\,dx = \int_0^{W'} \frac{U_{MAX}}{\cosh[q\text{å}(x-x_m)/kT]}\,dx$$

For normally doped Si with Kn~Kp (assumed) U_{MAX} is strongly peaked away from W, so we can extend the integral from W' → ∞.

Calculating U_{TOTAL}

$$U_{TOTAL} = \int_0^{\infty} \frac{U_{MAX}}{\cosh[q\text{å}(x-x_m)/kT]}\,dx \qquad \int_0^{\infty} \frac{dx}{\cosh[x]} = \frac{\Pi}{2}$$

After a change of variables, $U_{TOTAL} = \frac{D\,kT}{2q\text{å}} U_{MAX}$

Recognizing that: $\text{å} = \frac{V}{cm} = \frac{V_{bi}+V_{app}}{W}$

$$U_{TOTAL} = \frac{D}{2}\ \underbrace{\frac{kTW}{q(V_{bi}+V_{app})}}_{\text{ratio of thermal spreading to carrier positioning.}}\ U_{MAX}$$

ratio of thermal spreading
to carrier positioning.
Thermal Voltage

U_{TOTAL}

Substituting for U_{MAX} , we obtain

$$U_{TOTAL} = \underbrace{\frac{D}{2}\,\frac{kTW(N_T\text{ó}v)n_i}{2q(V_{bi}+V_{app})}}_{\text{"Jo" term}}\;\underbrace{Exp\left(-\frac{qV_{app}}{2kT}\right)}_{\text{Important}}$$

"Jo" term
Notice Vapp though

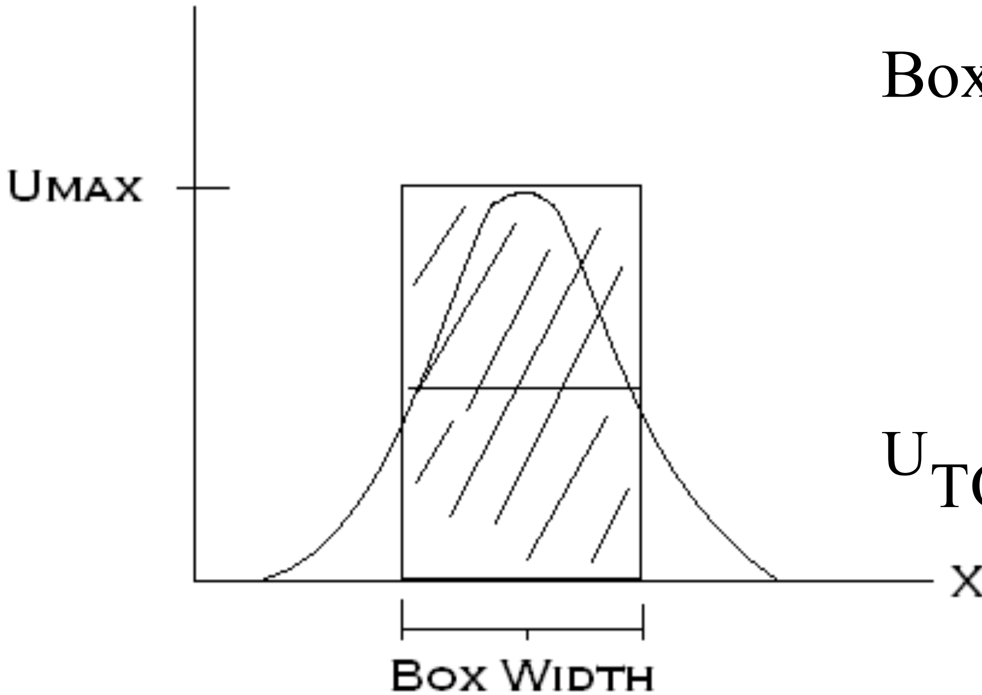
Important,
A = 2 not like
thermionic emission

A = 2: There are e^-s and h^+s recombining. It is *as if* you lose half of the voltage to the other carrier.

U_{TOTAL}: Everyone Does it Differently

Note: Nate and Mary quote a different value.

$$U_{TOTAL} = \int_0^{W'} U(x)\,dx = \text{Box Area}$$



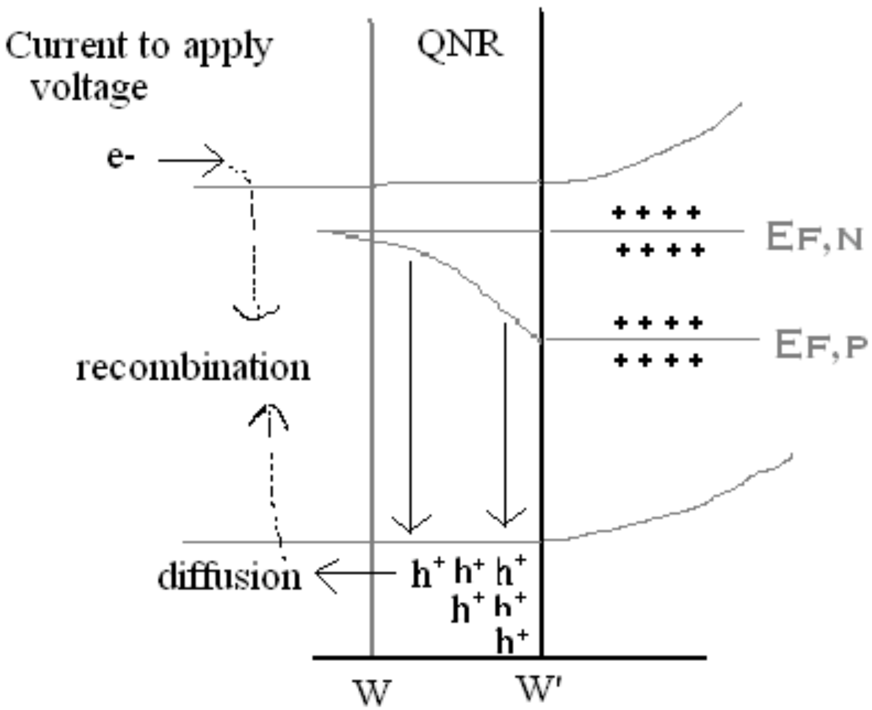
$$\text{Box Width} = W \underbrace{\frac{kT}{q(V_{bi}+V_{app})}}$$

Dimensionless: ratio thermal to applied volage

$$U_{TOTAL} = U_{MAX} W \frac{kT}{q} \frac{1}{(V_{bi}+V_{app})}$$

$$\rightarrow J_{dr} \text{ and } U_{TOTAL} \text{ off by } \frac{D}{2} : O(1)$$

Quasi-Neutral Region



Ionized donor atoms neutralized by injection of electrons into CB.

p(x) has not changed: before injection of electrons, p(w)<p(w’) despite flat bands

Concentration gradient causing holes to diffuse away from W’ into the bulk.

Need to understand Diffusion/Recombination/Generation equations

→ Assume No Generation

Diffusion-Recombination

Recombination-Diffusion = 0

Diffusion:

Fick's First Law: $\text{Flux} = -D_o \frac{\partial C_o(x,t)}{\partial x}$

Fick's Second Law: $\frac{\partial C_o(x,t)}{\partial t} = D_o \frac{\partial^2 C_o(x,t)}{\partial x^2}$

Putting it together:

$$\frac{\partial p}{\partial t} = 0 = \underbrace{\frac{p(x) - p_o(x)}{\tau_p}}_{\frac{\text{Excess Carrier}}{\text{lifetime}}} - D_o \frac{\partial^2 p(x)}{\partial x^2}$$

Need Boundary Conditions

Diffusion-Recombination

Assume no recombination in D.R. or at S.S.

$V_{app} < 0,$ so more holes at W' as $E_{F,p}$ suggest

$$\text{At } x=w'\text{:} \quad n(w')p(w') = n_i^2 \text{Exp}(-qV_{app}/kT)$$

$$n(w') = n_o \quad \text{since } E_{F,n} = E_{F,o}$$

Boundary Conditions for Bulk D-R stats:

$$p(w') = p_o \text{Exp}(-qV_{app}/kT)$$

$$p(\infty) = p_o$$

Diffusion-Recombination

After the math:

$$p(x) = p_o + p_o[\mathrm{Exp}(-qV_{app}/kT)-1]\mathrm{Exp}\left(-\frac{(x-w')}{L_p}\right)$$

$$L_p = \text{Diffusion Length} = \sqrt{D_p \hat{o}_p}$$

Does it work? D = R ? You tell me.

$$\text{recall, } D=D_o \frac{\partial^2 p(x)}{\partial x^2}$$

Diffusion-Recombination

$$\text{Current} = -q\,\text{Flux}_{hole} + q\,\text{Flux}_{electrons}$$

$$\text{Current} = -q\,\text{Flux}_{hole}\Big|_{W'} = -qD_p \frac{\partial p(x)}{\partial x}\Big|_{W'}$$

We only have p(x), so lets take region where flux only due to holes: at x=W'

$$\frac{\partial p}{\partial x} = -\frac{p_o}{L_p}[\text{Exp}(-qV_{app}/kT)-1]\,\text{Exp}\left(-\frac{(x-w')}{L_p}\right)$$

Evaluate at x=W'

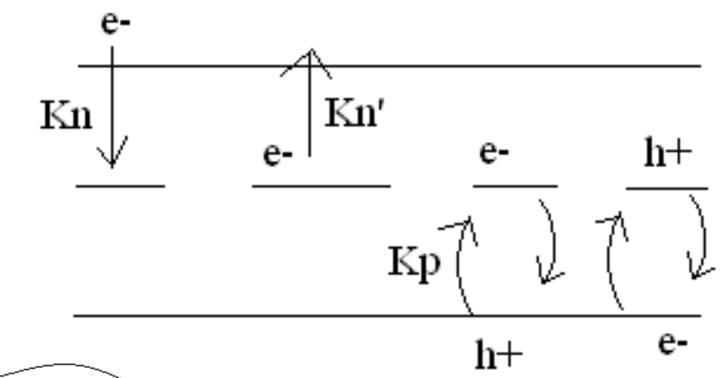
$$J_{B/R} = qD_p \frac{p_o}{L_p}[\text{Exp}(-qV_{app}/kT)-1] \;;\; n_o p_o = n_i^2,\; p_o = \frac{n_i^2}{n_o} \approx \frac{n_i^2}{N_D}$$

$$J_{B/R} = \underbrace{\frac{qD_p n_i^2}{L_p N_D}}_{J_o}[\text{Exp}(-qV_{app}/kT)-1]$$

Schockley-Read-Hall

$$U_{bulk} = N_T \frac{k_n k_p (n_b p_b - n_i^2)}{k_n (n_b + n_1) + k_p (p_b + p_1)}$$

$$U = \frac{-\partial n}{\partial t} = \frac{-\partial p}{\partial t}$$



$$\frac{-\partial n}{\partial t} = N_T k_n n_b (1 - f_T) - N_T k_n^{'} f_T \, (\text{D.O.S. in CB, empty})$$

$$\frac{-\partial p}{\partial t} = N_T k_p p_b f_T - N_T k_p^{'} (1 - f_T) (\text{D.O.S. in VB, filled})$$

$$k_n^{'} = k_n n_1, \quad k_p^{'} = k_p p_1$$

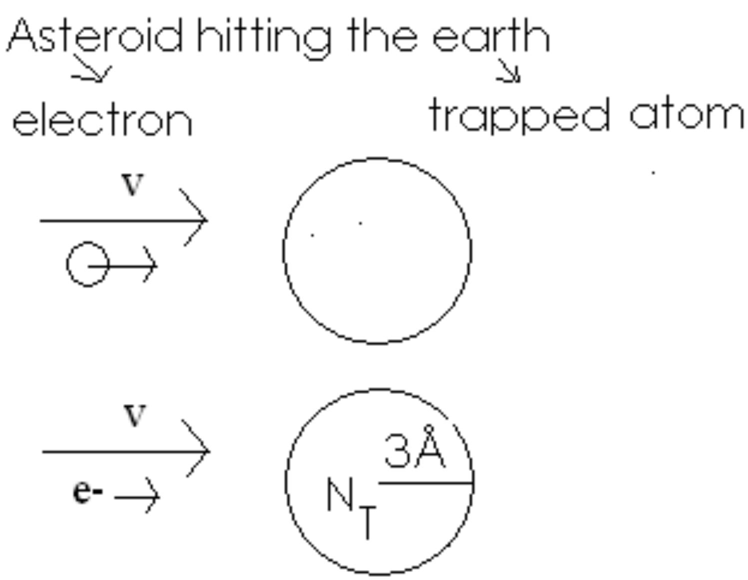
$$n_1 = N_C \text{Exp}[-(E_C - E_T)/kT]$$

$$p_1 = N_V \text{Exp}[-(E_T - E_V)/kT]$$

Trapped states can be caused by metal impurities, oxygen, or dopants

Schockley-Read-Hall

$$\text{Units for } U_{bulk} = cm^{-3}\frac{\frac{cm^6}{s^2}(cm^{-6})}{\frac{cm^3}{s^2}(cm^{-3})} = cm^{-3}s^{-1}$$



$$k = \text{ó}v = cm^2\frac{cm}{s}; \quad v_{th} = 10^7 cm/s;$$

$$\text{ó}_{trap} = \partial r^2 = \partial(3*10^{-8})^2 \approx 10^{-15}$$

$$k_n = v_{th}\text{ó}_{trap} \approx 10^{-8} cm^3/s$$

τ=lifetime of carrier

$$\hat{o}_n = \frac{1}{k_n N_T}$$

Schockley-Read-Hall

LLI: Lower Level Injection of Electrons and holes from photons

For n-type: $\ddot{A}n << n_b (10^{15})$

$\ddot{A}p << p_b (10^{5})$

10^{10}

HLI: Generate more electrons and holes than We had in our lattice:

$$10^{17}\text{-}10^{19} = \ddot{A}n = \ddot{A}p > n_b$$

$$n = \int_{E_C}^{\infty} f(\varepsilon)\mathrm{DOS};\ \text{ Using Boltzmann Statistics, } f(\varepsilon) = \mathrm{Exp}[-(E - E_F)/kT]$$

$$n = \ddot{A}n + n_{b,o}$$

$$p = \ddot{A}p + p_{b,o}$$

Schockley-Read-Hall

$$U_{bulk} = \frac{(\ddot{A}n+n_{b,o})(\ddot{A}p+p_{b,o}) - n_i^2}{\hat{o}_p(\ddot{A}n+n_{b,o}+n_1)+\hat{o}_n(\ddot{A}p+p_{b,o}+p_1)} = \frac{\ddot{A}n\ddot{A}p + p_{b,o}\ddot{A}n + n_{b,o}\ddot{A}p + \cancel{p_{b,o}n_{b,o} - n_i^2}}{\hat{o}_p(\ddot{A}n+n_{b,o}+n_1)+\hat{o}_n(\ddot{A}p+p_{b,o}+p_1)}$$

Assuming LLI of n-type

$$\underbrace{p_{b,o}}_{(10^5)} << \underbrace{\Delta n = \Delta p}_{(10^{10})} << \underbrace{n_{b,o}}_{(10^{15})}$$

U_{bulk} is dominated by $\Delta p * n_{b,o}$ term in numerator

and $\tau_p n_{b,o}$ in denominator

Leaving us with : $U_{bulk} = \frac{\ddot{A}p}{\hat{o}_p}$

We assumed $\hat{o}_n \approx \hat{o}_p$

Schockley-Read-Hall

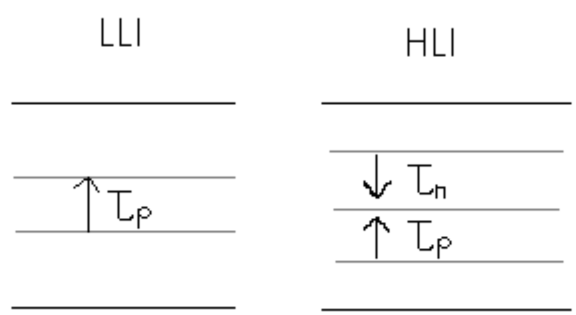
$$\text{LLI:}\quad U_{bulk} = \frac{\Delta p}{\tau_p} = -\frac{\partial n}{\partial t} = -\frac{\partial p}{\partial t}$$

Solving yields: $$p = \Delta p \mathrm{Exp}(-t/\tau_p) + p_o$$

For HLI, same assumptions: $$\text{HLI:}\quad U_{bulk} = \frac{\Delta p}{2\tau_p}$$

HLI decays twice as slowly

LLI: Wait for hole carriers to recombine



HLI: hole & e- carriers must recombine → twice as long